ESSE 3610 – Geodetic Concepts

PROJECT 3

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Executive summary➔

This report provides insight into conformal mapping and UTM projections, focusing on how to perform map projections in practice, and how to apply the combined scale factor in practice. Specifically, this report dives into the derivations for metric tensors and the applications of these derivations, the calculations required to determine the coordinates on a UTM plane, and the definition and applications of the combined scale factor.

The first section, Part A, focuses on a mathematical approach to understanding the metric tensor. This section includes deriving the metric tensor from geocentric coordinates, and then referencing the result to define and describe key aspects of the universal transverse Mercator, and the associated projections.

In Part B, the first task requires computing universal transverse Mercator coordinates, given geodetic coordinates for a certain reference ellipsoid. In the second task, we start with the universal transverse Mercator projection coordinates, and we need to work in the opposite direction as the first task to solve for the geodetic coordinates. The second task also requires the determination of the scale factor at the coordinates of the station given.

The third section, Part C, revolves more around the understanding and definition of what the combined scale factor is, how it is derived, and why it is important. In this section, it is required that formulas are provided for calculating the combined scale factor, assuming certain initial parameters such as the geodetic coordinates, and semi-axes for the reference ellipsoid.

The final section of the lab, Part D, expands on the second section, and requires the calculation of the combined scale factor, assuming the parameters previously defined. Then further building upon the results for section 2, it is required to determine the combined scale factor for different ellipsoidal heights.

Part A

Introduction

A metric tensor is a type of function that maps input vectors (or points) to a distance in geometric space. If the input vectors to the metric tensor are orthogonal, then the inner product of the tensor equates to 0. If the magnitude of resultant distance from the metric tensor varies from place to place, then the space is curved and is called a Riemannian space.

Methodology

a) To derive the metric tensor, begin by looking at the geocentric coordinates as a reference X, Y, Z.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ \left(N\frac{b^2}{a^2}+h\right)\sin\varphi \end{bmatrix}$$

Next, take the partial derivative with respect to each variable (h, ϕ, λ) and then apply the following equation:

$$T_{ij} = \delta_{ij}\frac{dx_i}{du_p} + \frac{dx_j}{du_s}$$

Where T_{ij} is the metric tensor for the geodetic coordinates, X_1 = X, X_2 = Y and X_3 = Z, U_1 = h, U_2 = ϕ, and U_3 = λ.

Also note that δ_{ij} is the Kronecker delta and is defined as follows:

$$\delta_{ij} = \begin{cases} 1 \; if \; i = j \\ 0 \; if \; i \neq j \end{cases}$$

Expanding the previous equation, the following obtain:

$$\left(\frac{dx}{dh}\right)^2 + \left(\frac{dy}{dh}\right)^2 + \left(\frac{dz}{dh}\right)^2$$

$$\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2 + \left(\frac{dz}{d\phi}\right)^2$$

$$\left(\frac{dx}{d\lambda}\right)^2 + \left(\frac{dy}{d\lambda}\right)^2 + \left(\frac{dz}{d\lambda}\right)^2$$

The above terms constitute the metric tensor for geodetic coordinates. Evaluating the terms, we obtain:

$$\left(\frac{dx}{dh}\right)^2 + \left(\frac{dy}{dh}\right)^2 + \left(\frac{dz}{dh}\right)^2 = \cos^2\varphi\cos^2\lambda + \cos^2\varphi\sin^2\lambda + \sin^2\varphi = 1$$

Since this is the partial derivative with respect to h, any terms with the variable N are neglected.

$$\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2 + \left(\frac{dz}{d\phi}\right)^2 = (-(N+h)\sin\varphi\cos\lambda)^2 + (-(N+h)\sin\varphi\sin\lambda)^2 + \left(\left(N\frac{b^2}{a^2}+h\right)\cos\varphi\right)^2$$

Next, consider M:

$$M = \frac{a(1-e^2)}{(1-e^2\sin^2\phi_i)^{\frac{3}{2}}}$$

Which can be re-written as:

$$M = \mathrm{N}^3\frac{\mathrm{b}^2}{\mathrm{a}^4}$$

To prove the above equation, multiply N by $N^2(b^2/a^4)$:

$$N^3 = \frac{a^3}{(1-\sin\phi_i{}^2 e^2)^{\frac{3}{2}}}$$

$$N^3\frac{b^2}{a^4} = \frac{a^3}{(1-\sin\phi_i{}^2 e^2)^{\frac{3}{2}}}\frac{b^2}{a^4} = \frac{a}{(1-\sin\phi_i{}^2 e^2)^{\frac{3}{2}}}\frac{b^2}{a^2}$$

However, b^2/a^2 is equivalent to:

$$\frac{b^2}{a^2} = 1 - e^2$$

Therefore:

$$N^3\frac{b^2}{a^4} = \frac{a}{(1-\sin\phi_i{}^2 e^2)^{\frac{3}{2}}}\frac{b^2}{a^2} = \frac{a(1-e^2)}{(1-\sin\phi_i{}^2 e^2)^{\frac{3}{2}}} = M$$

Now, (-N - h) and (Nb^2/a^2 + h) can be written as:

$$M + \mathrm{h}$$

Substituting M into the equation:

$$\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2 + \left(\frac{dz}{d\phi}\right)^2 = \big((M+h)\sin\varphi\cos\lambda\big)^2 + \big((M+h)\sin\varphi\sin\lambda\big)^2 + \big((M+h)\cos\varphi\big)^2 = (M+h)^2\,(\sin^2\varphi\cos^2\lambda + \sin^2\varphi\sin^2\lambda + \cos^2\varphi)$$

$$\sin^2\varphi\cos^2\lambda + \sin^2\varphi\sin^2\lambda + \cos^2\varphi = \sin^2\varphi\,(\cancel{\cos^2\lambda + \sin^2\lambda}) + \cos^2\varphi$$

Therefore:

$$\sin^2\varphi\,(1) + \cos^2\varphi = 1$$

Finally:

$$\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2 + \left(\frac{dz}{d\phi}\right)^2 = (M+h)^2$$

Last term of the metric tensor can be found by taking the partial derivative with respect to λ:

$$\left(\frac{dx}{d\lambda}\right)^2 + \left(\frac{dy}{d\lambda}\right)^2 + \left(\frac{dz}{d\lambda}\right)^2 = (-(N+h)\cos\varphi\sin\lambda)^2 + \big((N+h)\cos\varphi\cos\lambda\big)^2 + 0$$
$$= (N+h)^2\cos^2\varphi\sin^2\lambda + (N+h)^2\cos^2\varphi\cos^2\lambda$$
$$= (N+h)^2(\cos^2\varphi\sin^2\lambda + \cos^2\varphi\cos^2\lambda)$$

Where:

$$\cos^2\varphi\sin^2\lambda + \cos^2\varphi\cos^2\lambda = \cos^2\phi\,(\cancel{\sin^2\lambda + \cos^2\lambda}) = \cos^2\phi$$

Therefore:

$$\left(\frac{dx}{d\lambda}\right)^2 + \left(\frac{dy}{d\lambda}\right)^2 + \left(\frac{dz}{d\lambda}\right)^2 = (N+h)^2\cos^2\phi$$

Putting all the terms together, we obtain the following metric tensor for the geodetic coordinates (ϕ,λ,h):

$$T_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (M+h)^2 & 0 \\ 0 & 0 & (N+h)^2\cos^2\phi \end{bmatrix}$$

Note that the metric tensor is slightly different from the one defined in class, however, due to the nature of diagonal tensors, the diagonal values can be rotated depending on how the placeholder variables, i and j, are defined.

b) To write the metric tensor in terms of ϕ and λ based on the previous derivation, the matrix will be as follows:

$$T_{ij} = \begin{bmatrix} M^2 & 0 \\ 0 & N^2\cos^2\phi \end{bmatrix}$$

The resulting matrix is only 2x2 since h is zero. This means that the first term, which is based on the partial derivative taken with respect to h, is zero.

c) Conformal projections ensure that the meridians and parallels are intersecting at 90°. This is the most important type of projection in geodesy since it is the result of conserving the local shape.

d) Isometric latitudes are used in constructing ellipsoidal models of the Mercator projections and Transverse Mercator Projection. These latitudes give equal distance displacements along the meridians and parallels at any point on the ellipsoid, hence the name isometric. They are defined by the following equation:

$$\psi(\phi) = \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right] + \frac{e}{2}\ln\left[\frac{1 - e\sin\phi}{1 + e\sin\phi}\right]$$

For a Transverse Mercator Projection, the meridians and parallels must be intersecting at 90° in which isometric latitudes ensure this conformal requirement is met.

e) Lambert conformal conic projection uses cones as its developable surfaces. For a secant projection, the distortion increases with distance from the two standard parallels. Used for regions with a E/W expanse.

Transverse Mercator Projection uses a cylinder as its developable surface. It is a Mercator projection rotated by 90° hence the name transverse. The projection is conformal and used for regions a N/S expanse. It is also used for topographic mapping and it is the basis for plane coordinate systems such as the UTM projection.

Processing Inputs

The inputs for this problem are defined by the vectors that represent the points on the given surface. The orthogonality of the input vectors directly affects the shape of the resultant tensor. That is, if the input vectors are orthogonal, then the inner product of the tensor is 0, resulting in a diagonal tensor. If the input vectors are not orthogonal, then the inner product of the tensor is not 0, and the tensor is not diagonal.

Processing Outputs & Analysis

Applying the methodology yields the following metric tensor:

$$T_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (M+h)^2 & 0 \\ 0 & 0 & (N+h)^2\cos^2\phi \end{bmatrix}$$

Note that the metric tensor is slightly different from the one defined in class, however, due to the nature of diagonal tensors, the diagonal values can be rotated depending on how the placeholder variables, i and j, are defined.

Part B

Introduction

An important part of geodesy is converting between geodetic coordinates, latitude and longitude, to universal transverse Mercator plane coordinates, and vise versa. The following methodology does exactly this for given geodetic coordinates or UTM coordinates, and a reference ellipsoid.

Methodology (question 2)

2) This section only shows how to compute the needed values. The answers are provided using MATLAB code in the processing outputs section of this report. The code will be provided in the appendix. Note: In the code, all angles are converted into radians.

   We are given the astronomical geodetic latitude and longitude, which have been calculated using lab group number 14:

$$\phi = 45.02593^0\ N$$

$$\lambda = 281.5282^0\ W$$

For λ, we must convert it into east, therefore:

$$\lambda = \ 360^0 - 281.5282^0 = 78.4715^0 E$$

Next, find the corresponding zone number:

$$Zone\ \# = \frac{180 - \ \lambda}{6} = \frac{180 - (-281.5282)}{6} = 76.9 \approx 77$$

Since there are only 60 zones, subtract (77) – (60) = 17

For zone number 17, the defining constants are:

$$\lambda_o = 81°$$

$$E_o = 500{,}00\ m$$

$$N_o = 0$$

$$\phi_o = 0$$

$$K_o = 0.9996$$

Because $\phi_o = 0$, zone-specific constants ω_o and S_o are equal to zero, and parameters for the GRS 80 ellipsoid are:

$$a = 6{,}378{,}137\ m$$

$$e^2 = 0.006694380023$$

$$r = 6{,}367{,}449.14577\ m$$

Next, we need the rectifying latitude constants for the GRS ellipsoid which are:

$$U_o = -0.005048250776$$

$$U_2 = 0.000021259204$$

$$U_4 = -0.000000111423$$

$$U_6 = 0.0000000000626$$

From the rectifying latitude constants, we can find rectifying latitude ω, as a function of ϕ using the following equation:

$$\omega = \phi + (\sin\phi\ cos\phi)\left(U_o + \cos^2\phi\left(U_2 + \cos^2\phi\,(U_4 + U_6\cos^2\phi)\right)\right)\ [m]$$

Afterwards, the meridian distance can be calculated using:

$$S = K_o\omega r\ [m]$$

Then calculate radius of the prime vertical using:

$$\mathrm{R} = \frac{\mathrm{K_o a}}{\sqrt{(1-\varepsilon^2(\sin\phi)^2)}}\ [\boldsymbol{m}]$$

And

$$\eta^2 = \frac{\varepsilon^2}{1-\varepsilon^2}\cos^2\phi$$

Finally, define required constants for the calculation of N and E:

$$A_1 = -R\ [m]$$

$$A_2 = \frac{1}{2}R\tan\phi\ \ [m]$$

$$A_3 = \frac{1}{6}(1-\tan^2\phi+\eta^2)$$

$$A_4 = \frac{1}{12}\left(5-\tan^2\phi+\eta^2(9+4\eta^2)\right)$$

$$A_5 = \frac{1}{120}\left(5-18\tan^2\phi+\tan^4\phi+\eta^2(14-58\tan^2\phi)\right)$$

$$A_6 = \frac{1}{360}\left(61-58\tan^2\phi+\tan^4\phi+\eta^2(270-330\tan^2\phi)\right)$$

$$A_7 = \frac{1}{5040}(61 - 479\tan^2\phi + 179\tan^4\phi - \tan^6\phi)$$

Define the last auxiliary quantity as: $L = \lambda - \lambda_o \cos\phi \;\; [rad]$

And lastly, calculate E and N using:

$$E = E_o + A_1 L\left(1 + L^2\left(A_3 + L^2(A_5 + L^2 A_7 L^2)\right)\right) \; [m]$$

$$N = S - S_o + N_o + A_2 L^2\left(1 + L^2(A_4 + A_6 L^2)\right) \; [m]$$

Processing Inputs (question 2)

The inputs for solving this problem are relatively straightforward. These consist of the zone number which was determined prior, the scale factor, the false northing and easting, the geodetic coordinates, the reference ellipsoid parameters, and the reference ellipsoid constraints.

Processing Outputs & Analysis (question 2)

The results of the MATLAB code for question 2 are as follows. These values represent the universal transverse Mercator plane coordinates which correspond to the given geodetic coordinates. By inspection, these values seem correct since they are of the same order of magnitude as UTM coordinates should be for this zone.

E= 6.9920e+05 m

N= 4.9889e+06 m

Methodology (question 3)

Given plane coordinates on the UTM projection:

$$Zone = 17$$

$$N = 5107043.021\, m$$

$$E = 727960.546\, m$$

The projection constants and parameters and terms for the ellipsoid are:

$$\lambda_o = 81^0$$

$$E_o = 500{,}000$$

$$N_o = 0$$

$$\phi_o = 0$$

$$S_o = 0$$

$$K_o = 0.9996$$

$$a = 6{,}378{,}137\ m$$

$$e^2 = 0.006694380023$$

$$r = 6{,}367{,}449.14577$$

$$V_o = 0.005022893948$$

$$V_2 = 0.000029370625$$

$$V_4 = 0.000000235059$$

$$V_6 = 0.000000002181$$

Calculate the rectifying latitude ω using the following equation:

$$\omega = \frac{N - N_o + S_o}{K_o r}\ [rad]$$

Afterwards, calculate the foot point latitude ϕ' using the following equation:

$$\phi' = \omega + \sin\omega\cos\omega\,\{V_o + \cos^2\omega\,[V_2 + \cos^2\omega\,(V_4 + V_6\cos^2\omega)]\}\ [rad]$$

Next, compute $\Delta\phi$ using the following equation:

$$\Delta\phi = t_1\left[-\frac{\left(\frac{X'}{K_o}\right)^2}{2\,R_1\,N_1} + \frac{\left(\frac{X'}{K_o}\right)^4}{24R_1\,N_1^3}(5 + 3t_1^2)\right]\ [rad]$$

For the above equation, the corresponding variables are defined as:

$$R_1 = \frac{a(1 - \varepsilon^2)}{(1 - \varepsilon^2\sin^2\phi')^{\frac{3}{2}}}\ [m]$$

$$N_1 = \frac{a}{\sqrt{1 - \varepsilon^2\sin^2\phi'}}\ [m]$$

$$t_1 = \tan\phi'$$

$$\eta_1^2 = \frac{\varepsilon^2}{(1-\ \varepsilon^2)}\cos^2\phi'$$

$$X' = E -\ E_o[m]$$

After calculating all the above variables, $\Delta\phi$ can be calculated. After calculating $\Delta\phi$, ϕ can be found using:

$$\phi = \ \phi' +\ \Delta\phi\ [rad]$$

To find λ, we need to find $\Delta\lambda$ first using the following equation:

$$\Delta\lambda = \sec\phi'\left[\frac{X'}{N_1} - \frac{1}{6}\left(\frac{X'}{N_1}\right)^3 (1 + 2t_1^2 +\ \eta_1^2) + \frac{1}{120}\left(\frac{X'}{N_1}\right)^5 (5 + 28t_1^2 + 24t_1^4)\right]\ [rad]$$

After finding $\Delta\lambda$, we can find λ using the following equation:

$$\lambda = \ \lambda_o +\ \Delta\lambda\ [rad]$$

To find meridian convergence, use the following equation:

$$\gamma = D_1Q\big(1 + Q^2(D_3 + D_5Q^2)\big)\left(\frac{180}{\pi}\right)\ \ [rad]$$

For the above equation, the following are the corresponding variables needed to solve it:

$$D_1 = \tan\phi_f$$

$$D_3 = -\frac{1}{3}(1 + \tan^2\phi - \eta - 2\eta^2)$$

$$D_5 = \frac{1}{15}(2 + 5\tan^2\phi + 3\tan^4\phi)$$

$$Q = \frac{E -\ E_o}{R_f} = \frac{E - E_o}{N_1K_o}$$

After find the needed variables, the meridian convergence can be calculated. To find the scale factor, use the following equation:

$$k = \ K_o\big(1 + G_2Q^2(1 + G_4Q^2)\big)$$

For the above equation, the following are the corresponding variables needed to solve it:

$$G_2 = \frac{1}{2}(1 + \eta^2)$$

$$G_4 = \frac{1}{12}(1 + 5\eta^2)$$

After finding the needed variables, the scale factor can be calculated.

Processing Inputs (question 3)

This question is the reverse of the previous question, except with different values. Therefore, the inputs of this question are the same as the outputs of the previous question. These consist of the zone number, the universal transverse Mercator plane coordinates, and the reference ellipsoid parameters.

Processing Outputs & Analysis (question 3)

The results from the MATLAB code are as follows. These values represent the geodetic coordinates, the meridian convergence, and the scale factor at this point. By inspection, these values seem correct. We know that the zone in we are working in is the same zone as what was determined in the previous problem. Since we are in the same zone, the geodetic coordinates should be relatively similar to the given coordinates in question 2, which we can see they are.

$$\boldsymbol{\phi = 46.0789^o\ N}$$

$$\boldsymbol{\lambda = 281.9482^o\ W}$$

$$\boldsymbol{\gamma = 2.1245^o}$$

$$\boldsymbol{k = 1.0002}$$

Software Structure

The software structure of the code is very straightforward. It is a script with no intermediary functions used to calculate the required outputs for the questions. The full code can be viewed in the appendix.

Part C

Introduction

The combined scale factor is the combination of the average scale factor and the ellipsoidal correction factor. This combined scale factor is used when horizontal distances on a projection need to be reduced to grid distances. In simple terms, the combined scale factor can be represented as follows.

$$Combined\ Scale\ Factor = \bar{k}(\frac{R}{R+h})$$

Where $\bar{k}$ is the average scale factor at a point, R is the average radius of the Earth, and h is the average height above the ellipsoid of the path. The ellipsoidal correction factor, also known as the elevation factor, transforms the surface of the Earth onto the surface of an ellipsoid by measuring the height difference. Thus, the combined scale factor ultimately transforms the surface of the Earth to a map project by first transforming to an ellipsoid, then using the scale factor, transforms to the map projection.

Methodology

Given mean radius of earth through the following equation:

$$R = \frac{2a+b}{3}$$

Where a and b are semi-major and semi-minor axis respectively. To compute the elevation factor, use the following equation:

$$Elevation\ Factor = \frac{R}{R+h_{avg}}$$

Where R is the mean radius of earth and h is the ellipsoidal height. Next, use the following equation to compute the point scale factor, K:

$$k = K_o\left(1 + G_2Q^2(1 + G_4Q^2)\right)$$

Where K_o is a projection constant for the ellipsoid at a specific zone. G_2 and G_4 are:

$$G_2 = \frac{1}{2}(1+\eta^2)$$

$$G_4 = \frac{1}{12}(1+5\eta^2)$$

And η^2 can be calculated using the following equation:

$$\eta_1^2 = \frac{\varepsilon^2}{(1 - \varepsilon^2)} \cos^2 \phi'$$

Where ε^2 is a projection constant for the ellipsoid at a specific a zone and ϕ' is foot point latitude Finally, Q can be calculated using the following equation:

$$Q = \frac{E - E_o}{N_1 K_o}$$

Therefore, the combined scale factor is simply:

$$Combined\ scale\ factor = \frac{R}{R + h_{avg}} * K_o\left(1 + G_2 Q^2 (1 + G_4 Q^2)\right)$$

Processing Inputs

For this problem the inputs consist of arbitrary geodetic coordinates, ϕ, λ, and h which reside on the surface of a reference ellipsoid with semi-axes a and b. The specific datum used for this problem is the North American datum 83 (NAD83).

Processing Outputs & Analysis

The output for this section is the combined scale factor which is defined as follows

$$Combined\ scale\ factor = \frac{R}{R + h_{avg}} * K_o\left(1 + G_2 Q^2 (1 + G_4 Q^2)\right)$$

The term $K_o\left(1 + G_2 Q^2 (1 + G_4 Q^2)\right)$ is an expansion that represents the average scale factor at a point. Here, Q is the auxiliary, and G2 and G4 are scale factor coefficients.

Part D

Introduction

This section is an expansion for Part B, where for the same initial values, the scale factor is determined. Then, expanding further, the scale factor is calculated for increases in height in the ellipsoid.

Methodology

5a)

We are given the astronomical geodetic latitude and longitude, which have been calculated using lab group number 14:

$$\phi = 45.02593^0\ N$$

$$\lambda = 281.5282^0\ W$$

For λ, we must convert it into east, therefore:

$$\lambda = 360^0 - 281.5282^0 = 78.4715^0\ E$$

Using the point scale factor that was determined prior:

$$k = 1.0002$$

And the elevation scale factor which is defined as:

$$Elevation\ Factor = \frac{R}{R + h_{avg}}$$

The combined scale factor can be calculated according to:

$$Combined\ Scale\ Factor = \bar{k}(\frac{R}{R + h})$$

Where:

$$h = 0\ m, 50m, 100m, 150m, and\ 200m$$

A basic MATLAB script has been created to calculate the value of the combined scale factor.

Processing Inputs

The inputs for this problem are straightforward. The inputs consist of the scale factor, which is determined prior, and the elevation factor, which is a ratio of the average radius of the Earth (6371 km) and the average radius of the Earth plus the height above the ellipsoid. For a height of 0m above the ellipsoid, the ratio becomes 1:1 which means that the combined scale factor is the same as the scale factor at the point.

Processing Outputs & Analysis

For part a, the combined scale factor is 1.0002. This is the same as the scale factor because at an elevation of 0m above the ellipsoid, the elevation factor equates to 1. Ultimately this means that for the case of no elevation, the combined scale factor is equal to the scale factor at a point, k.

For part b, the following table shows the different combined scale factors at different elevations above the ellipsoid.

Height (m)	Combined Scale Factor (unitless)
50	1.000192150430463
100	1.000184300984132
150	1.000176451661003
200	1.000168602461075

As the elevation above the ellipsoid increases, the projection becomes more and more skewed. As the elevation approaches infinity, the elevation factor, which is a ratio of $\frac{R}{R+h}$ approaches 0. Since the combined scale factor is a product of the elevation factor and the average scale factor at a point, the result of an increasing elevation means the combined scale factor will decrease, as seen in the above trend.

Software Structure

The software structure of the code is very straightforward. It is a script with no intermediary functions used to calculate the required outputs for the questions. The full code can be viewed in the appendix.

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Appendix

Matlab code for Part B, 2

```
%Parameters

zone =  17; % zone number

K_not = 0.9996;% Scale factor

N_not = 0; % False northing

E_not = 500000; % False easting

phi_not = 0; % latitude of the grid origin 0 deg

lambda_not = 1.413716694; % longitude of the grid origin 81 deg

% GRS ellipsoid  parameters
a = 6378137; % semi major axis
ecen = 0.006694380023; % eccentricity
r = 6367449.14577; %

% Ellipsoid constants
u0 = -0.005048250776;
u2 = 0.000021259204;
u4 = -0.000000111423;
u6 = 0.000000000626;

%latitude and longitude
%phi  = 0.077559219;%latitude
%lambda = 4.893237842;%longtitude
phi = (45 + (1/60) + (31.3386/3600))*(pi/180);
lambda = (360 - (281 +(31/60)+(42.7553/3600)))*(pi/180);

% Rectifying latitude
omega = phi + (sin(phi)*cos(phi))*(u0 + (cos(phi))^2*(u2 + (cos(phi))^2*(u4 +
u6*(cos(phi))^2)));

% Meridian distance
s = K_not*omega*r;
s_not = 0;

% Radius of curvature
R = (K_not*a) / sqrt(1 - (ecen)*(sin(phi))^2);
zeta = (ecen / (1-ecen))*(cos(phi)^2) ;

%defining terms R1 to R7
A1 = -R;
A2 = (1/2)*R*tan(phi);
A3 = (1/6)*(1 - ((tan(phi))^2) + zeta);
A4 = (1/12)*(5 - ((tan(phi))^2) + zeta*(9 + (4*zeta)));
A5 = (1/120)*(5 - (18*((tan(phi))^2))+((tan(phi))^4) + zeta*(14 -
58*((tan(phi))^2)));
```

```
A6 = (1/360)*(61 - (58*((tan(phi))^2))+((tan(phi))^4) + zeta*(270-
330*((tan(phi))^2)));
A7 = (1/5040)*(61 - (479*((tan(phi))^2))+(179*((tan(phi))^4)) -
((tan(phi))^6));

% Last auxillary
l = (lambda - lambda_not)*cos(phi);

% Northing and Easting
N = s - s_not + N_not + A2*(l^2)*(1 + (l^2)*(A4 + A6*(l^2)))
E = E_not + A1*l*(1 + (l^2)*(A3 + (l^2)*(A5 + (l^2)*(A7*(l^2)))))
```

Matlab code for Part B, 3

```
%Given

N = 5107043.021; % Northing
E = 727960.546 ; %Easting

%Parameters
zone =  17; % zone number

K_not = 0.9996;% Scale factor

N_not = 0; % False northing

E_not = 500000; % False easting

phi_not = 0; % latitude of the grid origin 0 deg

lambda_not = 1.413716694; % longitude of the grid origin 81 deg

S_not = 0;

% GRS ellipsoid  parameters
a = 6378137; % semi major axis
ecen = 0.006694380023; % eccentricity
r = 6367449.14577; %

% Ellipsoid constants
V0 = 0.005022893948;
V2 = 0.000029370625;
V4 = 0.000000235059;
V6 = 0.000000002181;

% Rectifying lat
Omega = (N - N_not + S_not) / (K_not*r);

% footpoint latitude
phi_foot = Omega + (sin(Omega)*cos(Omega))*(V0 + (cos(Omega)^2)...
    *(V2 + (cos(Omega)^2)*(V4 +V6*(cos(Omega)^2))));

% radius of curvature
R = (K_not*a) / (1 - ecen*(sin(phi_foot).^2))^0.5;

% auxillary
Q = (E - E_not) / R;

% Zeta
zeta = (ecen / (1 - ecen))*(cos(phi_foot)^2);
% A2 to A7 terms
B2 = (-1/2)*tan(phi_foot)*(1 + zeta);
B3 = (-1/6)*(1 + 2*(tan(phi_foot)^2) + zeta);
B4 = (-1/12)*(5 + 3*(tan(phi_foot)^2) +...
    zeta*(1 - 9*(tan(phi_foot)^2)) - 4*zeta);
B5 = (1/120)*(5 + 28*(tan(phi_foot)^2) + 24*(tan(phi_foot)^4) +...
```

```
    zeta*(6 + 8*(tan(phi_foot)^2)));
B6 = (1/360)*(61 + 90*(tan(phi_foot)^2) + 45*(tan(phi_foot)^4) + ...
    zeta*(46 - 252*(tan(phi_foot)^2) - 90*(tan(phi_foot)^4)));
B7 = (-1/5040)*(61 + 662*(tan(phi_foot)^2) + 1320*(tan(phi_foot)^4)...
    + 720*(tan(phi_foot)^6));
% L
L = Q*(1 + (Q^2)*(B3 + (Q^2)*(B5 + B7*(Q^2))));

% Geodetic Latitude and Longitude
phi = (phi_foot + B2*(Q^2)*(1 + (Q^2)*(B4 + B6*(Q^2))))*(180 / pi);
lambda = (lambda_not - L / cos(phi_foot))*(180 / pi);
phi = phi
lambda = (360 - lambda)

% Convergence Terms
D1 = tan(phi_foot);
D3 = (-1/3)*(1 + (tan(phi_foot)^2) - zeta - 2*(zeta^2));
D5 = (1/15)*(2 + 5*(tan(phi_foot)^2) + 3*(tan(phi_foot)^4));
% Convergence
gama = (D1*Q*(1 + (Q^2)*(D3 + D5*(Q^2))))*(180 / pi);



% Scale Factor
G2 = (1/2)*(1 + zeta); % Scale factor term
G4 = (1/12)*(1 + 5*zeta); % Scale factor term

k = K_not*(1 + G2*(Q^2)*(1 + G4*(Q^2))) % scale factor
```

Matlab code for Part D, a

```
%latitude and longitude

phi = 45.02593;         %In radians
lambda = (360 - 281.5282);    %In radians
h = 0;    %In meters

%Mean radius of the earth
R = 6371000;     %In meters

%Elevation factor
E = R/(R+h);

%Point Scale factor
k=1.0002;

%combined scale factor
C = k*E
%in the command Window type "format long" to get the exact values
```

Matlab code for Part D, b

```
%In meters
h1 = 50;
h2 = 100;
h3 = 150;
h4 = 200;
%Mean radius of the earth
R = 6371000;     %In meters

%Elevation factor
E1 = R/(R+h1);
E2 = R/(R+h2);
E3 = R/(R+h3);
E4 = R/(R+h4);

%Point Scale factor
k=1.0002;

%combined scale factor
C1 = k*E1
C2 = k*E2
C3 = k*E3
C4 = k*E4
%in the command Window type "format long" to get the exact values
```